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# Isospin-breaking effects on $\alpha$ extracted in $B \rightarrow \pi\pi, \rho\rho, \rho\pi$

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## Abstract

Isospin-breaking in  $B \rightarrow \pi\pi$  caused by  $\pi^0 - \eta - \eta'$  mixing is studied in a model-independent way using flavor SU(3). Measured branching ratios for  $B^+ \rightarrow \pi^+\pi^0$ ,  $B^+ \rightarrow \pi^+\eta^{(\prime)}$  and  $B^0 \rightarrow \pi^0\eta^{(\prime)}$  imply an uncertainty in  $\alpha$  smaller than  $1.4^\circ$ . We find a negligible effect of  $\pi^0 - \eta - \eta'$  mixing on  $\alpha$  in  $B \rightarrow \rho\pi$ . Characterizing the effect of  $\rho^0 - \omega$  mixing in  $B \rightarrow \rho\rho$  and in  $B \rightarrow \rho\pi$  by the two-pion invariant mass dependence, we point out a way of constraining this effect experimentally or eliminating it altogether. We show that a model-independent shift in  $\alpha$  caused by electroweak penguin amplitudes in  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$ ,  $\Delta\alpha_{\text{EWP}} = (1.5 \pm 0.3)^\circ$ , may be slightly different in  $B \rightarrow \rho\pi$ . Other sources of isospin-breaking in these processes are briefly discussed.

## I. INTRODUCTIION

Isospin symmetry provides triangle relations for  $B \rightarrow \pi\pi$  and  $\bar{B} \rightarrow \pi\pi$ , which are governed by  $I = 0$  and  $I = 2$  amplitudes,

$$A_{+-} + \sqrt{2}A_{00} - \sqrt{2}A_{+0} = 0 , \quad A_{ij} \equiv A(B^0 \rightarrow \pi^i \pi^j) , \quad (1)$$

$$\bar{A}_{+-} + \sqrt{2}\bar{A}_{00} - \sqrt{2}\bar{A}_{-0} = 0 , \quad \bar{A}_{ij} \equiv A(\bar{B}^0 \rightarrow \pi^i \pi^j) . \quad (2)$$

These relations enable an extraction of the phase  $\alpha \equiv \phi_2 \equiv \text{Arg}(-V_{tb}^* V_{td} / V_{ub}^* V_{ud})$  from time-dependent CP asymmetries,  $S_{\pi\pi}$  and  $C_{\pi\pi}$ , in  $B^0(t) \rightarrow \pi^+ \pi^-$  [1]. The asymmetries determine  $\sin 2\alpha_{\text{eff}}$ ,

$$\sin(2\alpha_{\text{eff}}) = \frac{S_{\pi\pi}}{\sqrt{1 - C_{\pi\pi}^2}} . \quad (3)$$

The shift  $\Delta\alpha \equiv \alpha_{\text{eff}} - \alpha$  caused by the penguin amplitude is given by

$$\Delta\alpha \equiv \frac{1}{2} \text{Arg} (e^{2i\gamma} \bar{A}_{+-} A_{+-}^*) . \quad (4)$$

We define a measurable phase  $\Delta\alpha_0$ , given in terms of angles in the  $B$  and  $\bar{B}$  triangles,  $\phi \equiv \text{Arg}(A_{+-} A_{+0}^*)$ ,  $\bar{\phi} \equiv \text{Arg}(\bar{A}_{+-} \bar{A}_{-0}^*)$ :

$$\Delta\alpha_0 \equiv \frac{1}{2}(\bar{\phi} - \phi) = \frac{1}{2} [\text{Arg}(e^{2i\gamma} \bar{A}_{+-} A_{+-}^*) - \text{Arg}(e^{2i\gamma} \bar{A}_{-0} A_{+0}^*)] . \quad (5)$$

Neglecting very small electroweak penguin amplitudes which will be discussed below, a phase relation holds between the two  $\Delta I = 3/2$  tree amplitudes,

$$A_{+0} = e^{2i\gamma} \bar{A}_{-0} . \quad (6)$$

This implies  $\Delta\alpha = \Delta\alpha_0$ , fixing the relative orientation of the  $B$  triangle and the  $\bar{B}$  triangle (rotated by  $2\gamma$ ) such that these sides overlap. In this configuration  $\Delta\alpha$  is half the angle between sides corresponding to  $B^0 \rightarrow \pi^+ \pi^-$  and  $\bar{B}^0 \rightarrow \pi^+ \pi^-$ . This determines  $\Delta\alpha$  up to a fourfold ambiguity (including a sign ambiguity) related to the four possible relative orientations of the two triangles. In case that  $|A(B^0 \rightarrow \pi^0 \pi^0)|$  and  $|A(\bar{B}^0 \rightarrow \pi^0 \pi^0)|$  are not separately measured, while the charge-averaged neutral pion rate is measured, one may obtain upper bounds on  $|\Delta\alpha|$  [2].

The same method applies to polarization states in  $B^0(t) \rightarrow \rho^+ \rho^-$  which are CP-eigenstates, in particular to an even-CP longitudinally polarized state which is found to dominate this process [3]. A variant of isospin symmetry can also be used to learn  $\alpha$  in  $B^0(t) \rightarrow \rho\pi$  [4, 5]. Application of these methods to recent measurements by BaBar [3, 6] and Belle [7], where  $B \rightarrow \rho\rho$  played a dominant role, provides the currently most accurate direct determination of  $\alpha$  [8, 9],  $\alpha = (100^{+9}_{-10})^\circ$ . This precision can be improved [10] by resolving the sign ambiguity in  $\Delta\alpha$  under reasonably mild and testable assumptions about strong

phase differences between tree and penguin amplitudes in these processes. The error in  $\alpha$  is expected to be reduced further by improving the measurement of the direct CP asymmetry in  $B^0 \rightarrow \pi^0 \pi^0$  [11], or by using a prediction of a SCET analysis [12] that the phase difference between tree and color-suppressed amplitudes in  $B^+ \rightarrow \pi^+ \pi^0$  is small [13].

At this level of precision one is required to consider small electroweak penguin (EWP) amplitudes and corrections from isospin breaking caused by the  $u$  and  $d$  charge and mass differences. These corrections modify the geometry of the  $B$  and  $\bar{B}$  amplitude triangles. A model-independent study of electroweak penguin contributions in  $B \rightarrow \pi\pi$  was performed in [14]. Instead of overlapping with each other, the two sides of the two triangles,  $A_{+0}$  and  $e^{2i\gamma}\bar{A}_{-0}$ , were shown to form a calculable relative angle. Neglecting EWP operators with small Wilson coefficients ( $c_7$  and  $c_8$ ), isospin symmetry relates dominant  $\Delta I = 3/2$  EWP operators to  $\Delta I = 3/2$  current-current operators in the effective Hamiltonian, implying

$$\begin{aligned} (\Delta\alpha - \Delta\alpha_0)_{\text{EWP}} &\equiv \frac{1}{2}\text{Arg}(e^{2i\gamma}\bar{A}_{-0}A_{+0}^*) = -\frac{3}{2}\left(\frac{c_9 + c_{10}}{c_1 + c_2}\right)\frac{|V_{tb}V_{td}|}{|V_{ub}V_{ud}|}\sin\alpha \\ &= -\frac{3}{2}\left(\frac{c_9 + c_{10}}{c_1 + c_2}\right)\frac{\sin(\beta + \alpha)\sin\alpha}{\sin\beta} = +0.013\frac{\sin(\beta + \alpha)\sin\alpha}{\sin\beta}. \end{aligned} \quad (7)$$

The measured values of  $\beta$  and  $\alpha$  [8],  $\beta = (23.3 \pm 1.6)^\circ$ ,  $\alpha = (100^{+9}_{-10})^\circ$ , lead to a small calculable value with a negligible error,  $(\Delta\alpha - \Delta\alpha_0)_{\text{EWP}} = (1.5 \pm 0.3)^\circ$ . This shift must be included in the determination of  $\alpha$  by using  $\alpha = \alpha_{\text{eff}} - \Delta\alpha_0 - (\Delta\alpha - \Delta\alpha_0)_{\text{EWP}}$ .

Isospin-breaking due to nonzero  $u$  and  $d$  quark mass and charge differences has several effects on the analysis of  $B \rightarrow \pi\pi$ ,  $B \rightarrow \rho\rho$  and  $B \rightarrow \rho\pi$ . An important effect in  $B \rightarrow \pi\pi$ , caused by  $\pi^0 - \eta - \eta'$  mixing, was studied several years ago by Gardner [15] using generalized factorization [16]. She concluded that the resulting error on the extracted value of  $\alpha$  in the above range is about  $5^\circ$  including EWP contributions. The uncertainty may be even larger due to the approximation involved in this estimate. This would limit severely the future accuracy of determining  $\alpha$  in  $B \rightarrow \pi\pi$ . Gardner and Meissner [17] discussed briefly the appearance of a small  $\Delta I = 5/2$  amplitude in  $B \rightarrow \pi\pi$  which violates the isospin triangle relation. They mentioned isospin violation in  $B \rightarrow \rho\pi$  caused by  $\pi^0 - \eta - \eta'$  mixing and by  $\rho - \omega$  mixing, pointing out that the presence of an additional  $\Delta I = 5/2$  amplitude in  $B \rightarrow \rho\pi$ , involving the same weak phase as the tree amplitude, does not affect the isospin analysis. Refs. [18, 19, 20] studied direct CP violation in  $B^{+,0} \rightarrow (\pi^+\pi^-)_{\rho,\omega}\pi^{+,0}$  and in  $B^+ \rightarrow (\pi^+\pi^-)_{\rho,\omega}\rho^+$  caused by  $\rho - \omega$  mixing. These CP asymmetries affect the analyses of isospin related processes. Ref. [8] studied numerically the uncertainty in determining  $\alpha$  in  $B \rightarrow \rho\rho$ , assuming that isospin violating corrections in tree and penguin amplitudes in  $\sqrt{2}A(B^+ \rightarrow \rho^+\rho^0)$  are at a level of 4% relative to tree and penguin amplitudes in  $B^0 \rightarrow \rho^+\rho^-$ .

The purpose of this work is to analyze in a model-independent manner isospin-breaking effects, in particular the effects of  $\pi^0 - \eta - \eta'$  mixing and  $\rho - \omega$  mixing, on determining  $\alpha$  in  $B \rightarrow \pi\pi$ ,  $B \rightarrow \rho\rho$  and  $B \rightarrow \rho\pi$ . In Section II we will apply flavor SU(3) to  $B$  decays

into two charmless pseudoscalars, relating isospin-breaking terms in  $B \rightarrow \pi\pi$  to amplitudes of  $B \rightarrow \pi\eta$  and  $B \rightarrow \pi\eta'$ . Using measured rates, we will show that the effect of  $\pi^0 - \eta - \eta'$  mixing on determining  $\alpha$  in  $B \rightarrow \pi\pi$  is considerably smaller than estimated by Gardner. Turning in Section III to discuss the effects of  $\rho - \omega$  mixing on determining  $\alpha$  in  $B \rightarrow \rho\rho$ , we will show how to include this effect experimentally, without having to rely on a calculation of  $\rho - \omega$  mixing parameters. Section IV studies  $\pi^0 - \eta - \eta'$  mixing,  $\rho - \omega$  mixing and the effect of electroweak penguin amplitudes in  $B \rightarrow \rho\pi$ . In Section V we discuss briefly other sources for isospin-breaking, while Section VI concludes. Appendix A presents experimental constraints on parameters describing  $\rho - \omega$  mixing.

## II. EFFECT OF $\pi^0 - \eta - \eta'$ MIXING IN $B \rightarrow \pi\pi$

The mixing of  $\pi^0$  with  $\eta$  and  $\eta'$  introduces isospin-breaking in  $B \rightarrow \pi\pi$  through an additional  $I = 1$  amplitude, while the isospin conserving terms obey the triangle relation (1). We will use flavor SU(3) symmetry to estimate the isospin-breaking terms. SU(3) breaking corrections and smaller annihilation-like amplitudes which we neglect in these terms are higher order, and are expected to introduce an uncertainty at a level of 30%. A convenient way of applying flavor SU(3) to charmless  $B$  decays into two pseudoscalars is in terms of graphical representations describing flavor flow topologies [21, 22, 23, 24]. SU(3) amplitudes for two octets in the final state consists of a “tree” amplitude ( $t$ ) a “color-suppressed” amplitude ( $c$ ) a “penguin” amplitude ( $p$ ). Three annihilation-like amplitudes ( $a$ ,  $e$  and  $pa$ ) are expected to be much smaller [23, 25] and will be neglected. The remaining three amplitudes contain EWP contributions [26], the overall effect of which can be taken into account as summarized in Eq. (7). This effect can be included as explained above, and will therefore be disregarded in this section. For a singlet and an octet in the final state one has three SU(3) amplitudes [21], of which a “singlet penguin” amplitude ( $s$ ) dominates, while two annihilation-like amplitudes will be neglected [24].

We use quark content for mesons as in [23, 24], but a somewhat different phase convention:

$$\begin{aligned} B^0 &= -d\bar{b}, \quad B^+ = -u\bar{b}, \quad \pi_3 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad \pi^+ = -u\bar{d}, \quad \pi^- = d\bar{u}, \\ \eta &= \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - s\bar{s}), \quad \eta' = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + 2s\bar{s}) . \end{aligned} \quad (8)$$

The  $\eta$  and  $\eta'$  correspond to octet-singlet mixtures,

$$\begin{aligned} \eta &= \eta_8 \cos \theta - \eta_1 \sin \theta, \quad \eta' = \eta_8 \sin \theta + \eta_1 \cos \theta, \\ \eta_8 &= \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \quad \eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), \end{aligned} \quad (9)$$

with an “ideal” mixing angle  $\theta = \theta_0 = \sin^{-1}(-1/3) = -19.5^\circ$ . A slightly larger magnitude,  $\theta = -22^\circ$ , was used in [15], while a slightly smaller magnitude,  $\theta = (-15.7 \pm 1.7)^\circ$ , was

obtained in a very recent phenomenological fit [27]. While in the most part we use the value  $\theta_0$ , at the end of this section we discuss briefly the effect of a variation in  $\theta$ .

Expressions for decay amplitudes in terms of graphical SU(3) contributions, for final states involving pairs of isotriplet pions, and for pairs involving an isotriplet pion and  $\eta$  or  $\eta'$ , are obtained in a straightforward manner [23, 24, 28]:

$$\begin{aligned} A_{+-} &= t + p , & A_{33} &= \frac{1}{\sqrt{2}}(c - p) , & A_{+3} &= \frac{1}{\sqrt{2}}(t + c) , \\ A_{3\eta} &= \frac{1}{\sqrt{6}}(2p + s) , & A_{3\eta'} &= \frac{1}{\sqrt{3}}(p + 2s) , \\ A_{+\eta} &= \frac{1}{\sqrt{3}}(t + c + 2p + s) , & A_{+\eta'} &= \frac{1}{\sqrt{6}}(t + c + 2p + 4s) . \end{aligned} \quad (10)$$

The first three amplitudes for pure isotriplet pions obey clearly the isospin triangle relation (1). The purely  $\Delta I = 3/2$  amplitude  $A_{+3} = (t + c)/\sqrt{2}$  has a weak phase  $\gamma$ .

The mixing of  $\pi^0$ ,  $\eta$  and  $\eta'$  introduces a small isospin singlet component into the dominantly isotriplet neutral pion state,

$$|\pi^0\rangle = |\pi_3\rangle + \epsilon|\eta\rangle + \epsilon'|\eta'\rangle . \quad (11)$$

Values  $\epsilon = 0.014$ ,  $\epsilon' = 0.0077$  were used by Gardner [15], based on a calculation applying chiral perturbation theory [29]. We will take ranges of values as obtained in a recent update [30],  $\epsilon = 0.017 \pm 0.003$ ,  $\epsilon' = 0.004 \pm 0.001$ .

Neglecting terms quadratic in  $\epsilon$  and  $\epsilon'$ , we find decay amplitudes for the neutral pion state given by (11):

$$\begin{aligned} A_{+0} &= A_{+3} + \epsilon A_{+\eta} + \epsilon' A_{+\eta'} \\ &= \frac{1}{\sqrt{2}}(t + c)(1 + e_0) + \frac{1}{\sqrt{3}}\epsilon(2p + s) + \frac{\sqrt{2}}{\sqrt{3}}\epsilon'(p + 2s) , \end{aligned} \quad (12)$$

$$\begin{aligned} A_{00} &= A_{33} + \sqrt{2}\epsilon A_{3\eta} + \sqrt{2}\epsilon' A_{3\eta'} \\ &= \frac{1}{\sqrt{2}}(c - p) + \frac{1}{\sqrt{3}}\epsilon(2p + s) + \frac{\sqrt{2}}{\sqrt{3}}\epsilon'(p + 2s) , \end{aligned} \quad (13)$$

where

$$e_0 = \sqrt{\frac{2}{3}}\epsilon + \sqrt{\frac{1}{3}}\epsilon' = 0.016 \pm 0.003 . \quad (14)$$

We note the factors  $\sqrt{2}$  in the first line of Eq. (13). This takes into account final states of identical particles in  $A_{00}$  and  $A_{33}$  [31], compared to states which must be symmetrized in  $A_{3\eta}$  and  $A_{3\eta'}$ . Squares of amplitudes give decay rates when common phase space factors are implied.

Using these expressions, we find two important consequences of this amplitude decomposition which includes  $\pi^0 - \eta - \eta'$  mixing:

1. The triangle relation (1) is modified only slightly:

$$A_{+-} + \sqrt{2}A_{00} - \sqrt{2}A_{+0}(1 - e_0) = 0 . \quad (15)$$

2. The amplitude  $A_{+0}$  can be written in terms of the pure  $\Delta I = 3/2$  amplitude,  $A_{+3}$ , carrying a weak phase  $\gamma$ , corrected by isospin-breaking terms involving  $A_{0\eta}$  and  $A_{0\eta'}$ ,

$$A_{+0} = A_{+3}(1 + e_0) + \sqrt{2}\epsilon A_{0\eta} + \sqrt{2}\epsilon' A_{0\eta'} . \quad (16)$$

Our first conclusion is therefore that the physical  $B \rightarrow \pi\pi$  and  $\bar{B} \rightarrow \pi\pi$  decay amplitudes still obey triangle relations. The isospin-breaking factor,  $1 - e_0$ , multiplying the amplitude  $A_{+0}$  in (15), can be absorbed in this measurement. Since  $e_0$  is calculated to be between one and two percent, while the current error in  $|A_{+0}|$  is about 5% (see Eq. (20) below), the factor  $1 - e_0$  starts to play a non-negligible role and must be included in the construction of the isospin triangles [32]. The remaining error from the theoretical uncertainty in  $e_0$  given in (14) is only a fraction of a percent, causing a negligible error in determining  $\Delta\alpha_0$  from the angles in the two isospin triangles.

The second result, Eq. (16), implies that  $A_{+0}$  and its charge-conjugate no longer obey the exact phase relation (6). That is, since the weak phases of the small isospin-breaking terms in (16) differ from  $\gamma$ , the triangle (15) and the corresponding triangle for  $\bar{B}$  amplitudes rotated by an angle  $2\gamma$  do not share exactly a common base,  $A_{+0} \neq e^{2i\gamma} \bar{A}_{-0}$ . Denoting

$$\psi_{\eta'} \equiv \text{Arg} \left[ \frac{A_{0\eta'}}{A_{+0}} \right] , \quad \bar{\psi}_{\eta'} \equiv \text{Arg} \left[ \frac{\bar{A}_{0\eta'}}{\bar{A}_{-0}} \right] , \quad (17)$$

this introduces a change,  $\Delta\alpha - \Delta\alpha_0$ , given to first order in  $\epsilon$  and  $\epsilon'$  by

$$\begin{aligned} (\Delta\alpha - \Delta\alpha_0)_{\pi-\eta-\eta'} &\equiv \frac{1}{2} \text{Arg}(e^{2i\gamma} \bar{A}_{-0} A_{+0}^*) \\ &= \frac{1}{\sqrt{2}|A_{+0}|} \left[ \epsilon \left( |\bar{A}_{0\eta}| \sin \bar{\psi}_\eta - |A_{0\eta}| \sin \psi_\eta \right) + \epsilon' \left( |\bar{A}_{0\eta'}| \sin \bar{\psi}_{\eta'} - |A_{0\eta'}| \sin \psi_{\eta'} \right) \right] . \end{aligned} \quad (18)$$

Given that the phases  $\psi_{\eta'}$  and  $\bar{\psi}_{\eta'}$  are unknown, an immediate upper bound on  $|\Delta\alpha - \Delta\alpha_0|$  may be obtained by taking  $\bar{\psi}_{\eta'} = -\psi_{\eta'} = \pi/2$ :

$$\begin{aligned} |(\Delta\alpha - \Delta\alpha_0)_{\pi-\eta-\eta'}| &\leq \epsilon \left( \frac{|A_{0\eta}| + |\bar{A}_{0\eta}|}{\sqrt{2}|A_{+0}|} \right) + \epsilon' \left( \frac{|A_{0\eta'}| + |\bar{A}_{0\eta'}|}{\sqrt{2}|A_{+0}|} \right) \\ &\leq \sqrt{2 \frac{\tau_+}{\tau_0}} \left( \epsilon \sqrt{\frac{\mathcal{B}_{0\eta}}{\mathcal{B}_{+0}}} + \epsilon' \sqrt{\frac{\mathcal{B}_{0\eta'}}{\mathcal{B}_{+0}}} \right) . \end{aligned} \quad (19)$$

Here  $\mathcal{B}_{ij} \equiv (|A_{ij}|^2 + |\bar{A}_{ij}|^2)\tau_B/2$  denote charge-averaged branching ratios for corresponding decays, and  $\tau_+/\tau_0$  is the lifetime ratio of  $B^+$  and  $B^0$ . We neglect tiny corrections (at a level of a percent) in phase space factors.

Using world averaged values [33],

$$\frac{\tau_+}{\tau_0} = 1.081 \pm 0.015 , \quad \mathcal{B}_{+0} = (5.5 \pm 0.6) \times 10^{-6} \text{ [34, 35]} ,$$

$$\mathcal{B}_{0\eta} < 2.5 \times 10^{-6} \text{ (90\% CL) [36, 37]} , \quad \mathcal{B}_{0\eta'} < 3.7 \times 10^{-6} \text{ (90\% CL) [36]} , \quad (20)$$

we find at 90% CL

$$|(\Delta\alpha - \Delta\alpha_0)_{\pi-\eta-\eta'}| < 1.05\epsilon + 1.28\epsilon' = 1.6^\circ . \quad (21)$$

The phases  $\psi_\eta$  and  $\psi_{\eta'}$  may actually be measured within discrete ambiguities through two triangle relations implied by Eqs. (10), valid to zeroth order in  $\epsilon$  and  $\epsilon'$ ,

$$A_{+\eta} = \frac{\sqrt{2}}{\sqrt{3}} A_{+0} + \sqrt{2} A_{0\eta} ,$$

$$A_{+\eta'} = \frac{1}{\sqrt{3}} A_{+0} + \sqrt{2} A_{0\eta'} . \quad (22)$$

Measuring the magnitudes of the three amplitudes in each of the two triangles determines  $\cos\psi_\eta$  and  $\cos\psi_{\eta'}$ . Similar relations for charge-conjugate amplitudes determine  $\bar{\psi}_\eta$  and  $\bar{\psi}_{\eta'}$ . To determine separately  $|A_{0\eta'}|$  and  $|\bar{A}_{0\eta'}|$  would require measuring also CP asymmetries in these channels. In the absence of these asymmetry measurements, one may use charge-averaged rates alone to improve the upper bound (19). Maximizing  $(\Delta\alpha - \Delta\alpha_0)_{\pi-\eta-\eta'}$  in (18) by varying  $\psi_{\eta'}$  and  $\bar{\psi}_{\eta'}$ , while keeping  $\mathcal{B}_{+\eta'}$  and the upper bounds on  $\mathcal{B}_{0\eta'}$  fixed, we find that a maximum is obtained for  $|\bar{A}_{0\eta'}| = |A_{0\eta'}|$ :

$$|(\Delta\alpha - \Delta\alpha_0)_{\pi-\eta-\eta'}| \leq \sqrt{2 \frac{\tau_+}{\tau_0}} \left( \epsilon \sqrt{\frac{\mathcal{B}_{0\eta}}{\mathcal{B}_{+0}} (1 - r_\eta)} + \epsilon' \sqrt{\frac{\mathcal{B}_{0\eta'}}{\mathcal{B}_{+0}} (1 - r_{\eta'})} \right) , \quad (23)$$

where

$$r_\eta = \frac{3}{16} \frac{\left[ \sqrt{\frac{\tau_0}{\tau_+}} (\mathcal{B}_{+\eta} - \frac{2}{3} \mathcal{B}_{+0}) - 2 \sqrt{\frac{\tau_+}{\tau_0}} \mathcal{B}_{0\eta} \right]^2}{\mathcal{B}_{+0} \mathcal{B}_{0\eta}} , \quad r_{\eta'} = \frac{3}{8} \frac{\left[ \sqrt{\frac{\tau_0}{\tau_+}} (\mathcal{B}_{+\eta'} - \frac{1}{3} \mathcal{B}_{+0}) - 2 \sqrt{\frac{\tau_+}{\tau_0}} \mathcal{B}_{0\eta'} \right]^2}{\mathcal{B}_{+0} \mathcal{B}_{0\eta'}} . \quad (24)$$

Using world averaged values [33],

$$\mathcal{B}_{+\eta} = (4.8 \pm 0.6) \times 10^{-6} \text{ [37, 38]} , \quad \mathcal{B}_{+\eta'} = (4.2 \pm 1.1) \times 10^{-6} \text{ [39]} , \quad (25)$$

and values in (20), we find at 90% CL

$$|(\Delta\alpha - \Delta\alpha_0)_{\pi-\eta-\eta'}| < 1.4^\circ . \quad (26)$$

This is only a slight improvement relative to (21).

The upper bounds (21) and (26) involve an uncertainty of about 30% from SU(3) breaking and small annihilation amplitudes which we have neglected. The bounds are seen to be

considerably lower than the estimate of the uncertainty,  $\delta\alpha \sim 5^\circ$ , obtained in [15] using generalized factorization. These bounds may be tightened further by reducing errors in the relevant  $B^+$  decay branching ratios, and in particular by improving the upper limits on  $\mathcal{B}(B \rightarrow \pi^0\eta)$  and  $\mathcal{B}(B \rightarrow \pi^0\eta')$ . These experimental upper limits play also an important role in interpreting theoretically [40] the measured deviation of the time-dependent CP asymmetry in  $B^0 \rightarrow \eta'K_S$  from  $\sin 2\beta \sin \Delta m t$  [41]. This makes the case for their further improvement even stronger.

Our analysis was based on the “ideal” mixing angle,  $\theta_0 = \sin^{-1}(-1/3)$ , which we used in (8). Defining a general mixing angle,  $\theta = \theta_0 + \delta$ , one may show that for this case one must replace  $e_0$  in (15) and (16) by  $e$ ,

$$e = \sqrt{\frac{2}{3}}(\epsilon \cos \delta + \epsilon' \sin \delta) + \sqrt{\frac{1}{3}}(-\epsilon \sin \delta + \epsilon' \cos \delta) . \quad (27)$$

The small theoretical uncertainty in the value of  $\theta$  ( $|\delta| < 6^\circ$ ) implies a value for  $e$  within the uncertainty in  $e_0$  given in (14). Furthermore, the terms  $\epsilon$  and  $\epsilon'$  in (16) are preserved by replacing  $\theta_0$  by  $\theta$ , implying that the upper bounds (21) and (26) are unaffected by varying  $\theta$ .

### III. EFFECTS OF $\rho - \omega$ MIXING IN $B \rightarrow \rho\rho$

The processes  $B \rightarrow \rho^i(\pi_1\pi_2)\rho^j(\pi_3\pi_4)$  are quasi two-body decays involving four pions in the final state. To account for the  $\rho$  width, the two  $\rho$  mesons are defined by choosing suitable common ranges of invariant masses for the two-pion pairs,  $s_{12} \equiv (p_1 + p_2)^2$  and  $s_{34} \equiv (p_3 + p_4)^2$ . One uses the pion angular distributions in the  $\rho$  rest frames to project longitudinally polarized states which were shown to dominate  $B \rightarrow \rho\rho$  [3]. Applying the isospin analysis to  $B \rightarrow (\rho\rho)_{\text{long}}$  proceeds identically to  $B \rightarrow \pi\pi$  [1, 2] in the limit of a vanishing  $\rho$  width. (In principle, the method applies separately to each transversity state.) The  $\rho$  width has the effect that two  $\rho$  mesons with different invariant masses,  $s_{12} \neq s_{34}$ , cannot be considered identical. Therefore Bose symmetry does not exclude a final  $I = 1$  state [42], for which the amplitude is antisymmetric under  $s_{12} \leftrightarrow s_{34}$ . This amplitude does not interfere in the decay rate with the usual symmetric  $I = 0$  and  $I = 2$  amplitudes. The effect of the  $I = 1$  amplitude on the isospin analysis, of order  $(\Gamma_\rho/m_\rho)^2 \simeq 0.04$ , may be taken into account by including it in the fit. In principle, the effect may be eliminated by decreasing the width of the  $\rho$  band, however this would also decrease the statistics.

In the following we will disregard this  $I = 1$  term, which contributes also in the isospin symmetry limit, studying isospin-breaking effects of the same order. To make our point, consider first the general invariant mass dependence of decay amplitudes for the three distinct

charged  $\rho$  states,

$$\begin{aligned} A_{+-}(s_{12}, s_{34}) &\equiv A(B^0 \rightarrow (\pi^+ \pi^0)_{12} (\pi^0 \pi^-)_{34}) = A(B^0 \rightarrow \rho^+ \rho^-) f_c(s_{12}) f_c(s_{34}) , \\ A_{+0}(s_{12}, s_{34}) &\equiv A(B^+ \rightarrow (\pi^+ \pi^0)_{12} (\pi^+ \pi^-)_{34}) = A(B^+ \rightarrow \rho^+ \rho^0) f_c(s_{12}) f_n(s_{34}) , \\ A_{00}(s_{12}, s_{34}) &\equiv A(B^0 \rightarrow (\pi^+ \pi^-)_{12} (\pi^+ \pi^-)_{34}) = A(B^0 \rightarrow \rho^0 \rho^0) f_n(s_{12}) f_n(s_{34}) , \end{aligned} \quad (28)$$

where  $f_{c,n}$  are usually taken as Breit-Wigner factors. If isospin symmetry were exact, then  $f_n(s) = f_c(s)$ , so that the two ratios,

$$\frac{A_{+0}(s_{12}, s_{34})}{A_{+-}(s_{12}, s_{34})} \quad \text{and} \quad \frac{A_{00}(s_{12}, s_{34})}{A_{+-}(s_{12}, s_{34})} , \quad (29)$$

would be independent of  $s_{12}$  and  $s_{34}$  in the quasi two-body approximation. Any observed dependence of these ratios on the invariant masses would indicate either isospin-breaking, or dependence of  $A(B \rightarrow \rho^i \rho^j)$  on  $s_{12}$  and  $s_{34}$ . The latter possibility may be fitted experimentally by considering this dependence over the entire widths of the two  $\rho$  mesons [42]. We will study isospin-breaking in a narrow range of invariant masses defined by the narrow  $\omega$  resonance, for which  $A(B \rightarrow \rho^i \rho^j)$  may be assumed to be constant. Our purpose is to use the measured invariant mass dependence in (28) as a tool for extracting the isospin symmetric  $B \rightarrow \rho \rho$  amplitudes which obey a triangle relation similar to (1).

Let us now study  $\rho - \omega$  mixing following a formalism developed in [43]. The physical  $\rho$  and  $\omega$  fields are mixtures of an isovector field,  $\rho_I$ , and an isoscalar fields,  $\omega_I$ ,

$$\begin{aligned} \rho^0 &= \rho_I - \epsilon_1 \omega_I , \\ \omega &= \omega_I + \epsilon_2 \rho_I . \end{aligned} \quad (30)$$

The isospin-breaking parameters,  $\epsilon_{1,2}$ , are of order of a few percent. A precise knowledge of their magnitudes will not be needed (see appendix A for current experimental constraints), as they will be hidden in an isospin-breaking function to be introduced below. An expansion in  $\epsilon_{1,2}$  will be carried out to first order in these parameters.

Consider the transformation between the isospin basis and the physical basis for the scalar parts of the vector meson propagators. The mixed propagator in the isospin basis,  $D_{\rho\omega}^I \equiv \langle \rho_I \omega_I \rangle_0$ , has poles at the  $\rho$  and  $\omega$  masses, and is conventionally written in the form

$$D_{\rho\omega}^I(s) = \Pi_{\rho\omega}(s) D_{\rho\rho}(s) D_{\omega\omega}(s) . \quad (31)$$

The physical basis is defined by requiring that  $\Pi_{\rho\omega}$  does not have poles. The scalar parts of the physical propagators can be approximated near the poles by Breit-Wigner forms,

$$D_{VV}(s) = \frac{1}{s - m_V^2 + im_V \Gamma_V} , \quad V = \rho, \omega . \quad (32)$$

The values of  $\epsilon_{1,2}$  are chosen such that the mixed propagator in the physical basis,  $D_{\rho\omega} \equiv \langle \rho \omega \rangle_0$  has no poles,

$$D_{\rho\omega}^I = D_{\rho\omega} + \epsilon_1 D_{\omega\omega} - \epsilon_2 D_{\rho\rho} . \quad (33)$$

All three terms on the right-hand-side are of order  $\epsilon_{1,2}$ . The equalities,  $D_{VV}^I(s) = D_{VV}(s)$ , ( $V = \rho, \omega$ ), hold to first order in  $\epsilon_{1,2}$ . For instance, the second and third terms in the relation,

$$D_{\rho\rho}^I = D_{\rho\rho} + 2\epsilon_1 D_{\rho\omega} + \epsilon_1^2 D_{\omega\omega} , \quad (34)$$

are second order in  $\epsilon_{1,2}$  and will be neglected.

To introduce isospin-breaking most generally, we take for neutral and charged  $\rho$  mesons independent  $\rho \rightarrow \pi\pi$  couplings,  $g_I \equiv g(\rho_I \rightarrow \pi^+ \pi^-)$ ,  $g_c \equiv g(\rho^+ \rightarrow \pi^+ \pi_3)$ , and independent mass and width parameters entering  $D_{\rho\rho}^I$  and  $D_{\rho\rho}^c$ . We neglect higher order effects in  $g(\rho^+ \rightarrow \pi^+ \pi^0)$  caused by  $\pi^0 - \eta - \eta'$  mixing (11),

$$g(\rho^+ \rightarrow \pi^+ \pi^0) = g(\rho^+ \rightarrow \pi^+ \pi_3) + \epsilon g(\rho^+ \rightarrow \pi^+ \eta) + \epsilon' g(\rho^+ \rightarrow \pi^+ \eta') , \quad (35)$$

because the two couplings multiplying  $\epsilon$  and  $\epsilon'$  violate G-parity and are thus further suppressed; for instance [44]

$$\left| \frac{g(\rho^+ \rightarrow \pi^+ \eta)}{g(\rho^+ \rightarrow \pi^+ \pi_3)} \right| = \left[ \left( 1 - \frac{m_\eta^2}{m_\rho^2} \right) \frac{Br(\rho^+ \rightarrow \pi^+ \eta)}{Br(\rho^+ \rightarrow \pi^+ \pi_3)} \right]^{1/2} < 0.055 \quad (84\% \text{CL}) . \quad (36)$$

In the presence of isospin-breaking  $\omega_I$  couples to two-pions with a coupling  $g(\omega_I \rightarrow \pi\pi)$  of order  $\epsilon_i g(\rho_I \pi\pi)$ . The decay  $B \rightarrow (\pi\pi)^0 X$  then proceeds either through  $\rho_I$  or through  $\omega_I$ . Working to first order in isospin-breaking, these two contributions enter through a linear combination of the two propagators, both of order  $\epsilon_i$ ,

$$\tilde{D}_{\rho\omega}(s) \equiv D_{\rho\omega}^I(s) + \frac{g(\omega_I \rightarrow \pi\pi)}{g(\rho_I \rightarrow \pi\pi)} D_{\omega\omega}^I(s) . \quad (37)$$

Thus, one finds expressions for  $B$  decay amplitudes into four pions, including terms which are first order in  $\epsilon_i$ :

$$A_{+-}(s_{12}, s_{34}) = g_c^2 A(B^0 \rightarrow \rho^+ \rho^-) D_{\rho\rho}^c(s_{12}) D_{\rho\rho}^c(s_{34}) , \quad (38)$$

$$\begin{aligned} A_{+0}(s_{12}, s_{34}) = & g_c g_I \left[ A(B^+ \rightarrow \rho^+ \rho_I) D_{\rho\rho}(s_{34}) \right. \\ & \left. + A(B^+ \rightarrow \rho^+ \omega_I) \tilde{D}_{\rho\omega}(s_{34}) \right] D_{\rho\rho}^c(s_{12}) , \end{aligned} \quad (39)$$

$$\begin{aligned} A_{00}(s_{12}, s_{34}) = & g_I^2 \left[ A(B^0 \rightarrow \rho_I \rho_I) D_{\rho\rho}(s_{12}) D_{\rho\rho}(s_{34}) \right. \\ & \left. + \frac{1}{\sqrt{2}} A(B^0 \rightarrow \rho_I \omega_I) \left( \tilde{D}_{\rho\omega}(s_{12}) D_{\rho\rho}(s_{34}) + (s_{12} \leftrightarrow s_{34}) \right) \right] . \end{aligned} \quad (40)$$

An implicit angular dependence in (38)-(40), corresponding to given polarization states [45], is independent of  $s_{12}$  and  $s_{34}$ .

In the isospin symmetry limit,  $D_{\rho\rho}^c = D_{\rho\rho}$ ,  $\tilde{D}_{\rho\omega} = 0$ . Isospin-breaking is given by deviations from these equalities and, for the case of  $\rho - \omega$  mixing, is parametrized most generally by Eqs. (38)-(40). Each term in a given row has a distinct dependence on  $s_{12}$  and  $s_{34}$ , characterized near the  $\rho$  and  $\omega$  poles by (31), (32) and (37). Taking  $g_c/g_I = 1.005 \pm 0.010$  [44],

the invariant mass distributions of the three processes permit in principle a determination of the three magnitudes,  $|A(B^0 \rightarrow \rho^+ \rho^-)|$ ,  $|A(B^+ \rightarrow \rho^+ \rho_I)|$  and  $|A(B^0 \rightarrow \rho_I \rho_I)|$ , forming the isospin triangle,

$$A(B^0 \rightarrow \rho^+ \rho^-) + \sqrt{2}A(B^0 \rightarrow \rho_I \rho_I) - \sqrt{2}A(B^+ \rightarrow \rho^+ \rho_I) = 0 . \quad (41)$$

Once this triangle and its charge-conjugate are formed, one uses a phase relation for  $A(B^\pm \rightarrow \rho^\pm \rho_I)$  analogous to (6) and the CP asymmetry in  $B^0(t) \rightarrow \rho^+ \rho^-$  to determine  $\alpha$ . This then provides a way of extracting  $\alpha$  free of effects from  $\rho - \omega$  mixing. Electroweak penguin contributions are treated as in  $B \rightarrow \pi\pi$ , Eq. (7).

The extraction of the pure isospin amplitudes  $|A(B \rightarrow \rho\rho)|$  may be facilitated by using information from direct measurements of  $A(B^+ \rightarrow \rho^+ \omega)$  and  $A(B^0 \rightarrow \rho^0 \omega)$  entering the isospin-breaking terms in (38)-(40). Also, the isospin-breaking function  $\tilde{D}_{\rho\omega}(s)$  is the same as the one fitted to the pion form factor [46]. Denoting

$$\tilde{D}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(s) \frac{1}{[s - m_\rho^2 + im_\rho \Gamma_\rho]} \frac{1}{[s - m_\omega^2 + im_\omega \Gamma_\omega]} , \quad (42)$$

the fit yields  $\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300$  MeV<sup>2</sup>, involving a possible small imaginary part compatible with zero. The slope at  $s = m_\omega^2$ ,  $\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04$ , is consistent with zero. The exact  $s$  dependence of  $\tilde{\Pi}_{\rho\omega}$  is unimportant because its contribution is dominated by the narrow  $\omega$  width.

At the  $\omega$  mass this gives

$$\frac{|\tilde{D}_{\rho\omega}(m_\omega^2)|}{|D_{\rho\rho}(m_\omega^2)|} = 0.53 \pm 0.05 . \quad (43)$$

Using the experimental values [47, 48],

$$\mathcal{B}(B^+ \rightarrow \rho^+ \rho^0) = (26.4^{+6.1}_{-6.4}) \times 10^{-6} , \quad \mathcal{B}(B^+ \rightarrow \rho^+ \omega) = (12.6^{+4.1}_{-3.8}) \times 10^{-6} , \quad (44)$$

and neglecting possible CP asymmetries in these processes, leads to

$$\frac{|A(B^+ \rightarrow \rho^+ \omega) \tilde{D}_{\rho\omega}(m_\omega^2)|}{|A(B^+ \rightarrow \rho^+ \rho^0) D_{\rho\rho}(m_\omega^2)|} = 0.36 \pm 0.08 . \quad (45)$$

While this ratio becomes 0.02, typical for isospin-breaking, when weighed by the  $\omega$  and  $\rho$  widths, it has a large effect at the  $\omega$  mass.

In order to demonstrate the effect of  $\rho - \omega$  mixing on the  $\pi^+ \pi^-$  invariant mass distribution applying Eq. (39), one must use some information about the relative magnitudes and relative phases of the amplitudes for  $B^\pm \rightarrow \rho^\pm \rho^0$  and  $B^+ \rightarrow \rho^\pm \omega$ . While the former amplitudes are pure “tree” (we neglect very small EWP contributions), involving a single CKM phase  $\text{Arg}(V_{ub}^* V_{ud}) = \gamma$ , the latter involve also penguin contributions with weak phase  $\text{Arg}(V_{cb}^* V_{cd}) = \pi$  in the  $c$ -convention [49],

$$\sqrt{2}A(B^+ \rightarrow \rho^+ \rho^0) = t + c , \quad \sqrt{2}A(B^+ \rightarrow \rho^+ \omega) = t + c + 2p + 2s . \quad (46)$$

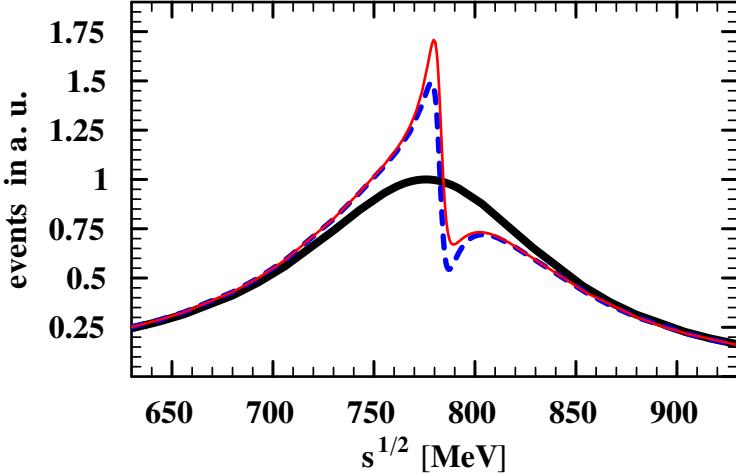


FIG. 1: Invariant mass distributions for  $\pi^+\pi^-$  in  $B^\pm \rightarrow \rho^\pm\pi^+\pi^-$  demonstrating  $\rho - \omega$  mixing, using hadronic parameters as given in the text. Dashed (blue) line represents  $B^+$  decays; solid (red) line represents  $B^-$  decays; thick (black) line describes a case neglecting  $\rho - \omega$  mixing.

For our purpose, the terms  $t, c, p$  and  $s$  represent SU(3) amplitudes for longitudinally polarized vector mesons, similar to those defined in Sec. II for two pseudoscalars. The amplitude  $s$  is OZI-suppressed and is expected to be negligible. (A similar amplitude in decays to a vector meson and a pseudoscalar meson dominates  $B^+ \rightarrow \phi\pi^+$  [50].) The ratio  $|p|/|t + c|$  is small, about 0.1, as can be inferred from the small branching ratio of  $B^0 \rightarrow \rho^0\rho^0$  [6], or (by flavor SU(3)) from the measured longitudinal branching fraction of  $B \rightarrow K^*\phi$  [33].

For illustration, we use  $2|s+p|/|t+c| = 0.2$ ,  $\gamma = 57^\circ$ , choosing the strong phase difference between  $s + p$  and  $t + c$  to be zero, so that the ratio  $\mathcal{B}(B^+ \rightarrow \rho^+\omega)/\mathcal{B}(B^+ \rightarrow \rho^+\rho^0) = 0.82$  is smaller than one as implied by experiment, Eq. (44). (Small transverse contributions are neglected.) These parameters determine relative magnitudes and relative phases between  $A(B^\pm \rightarrow \rho^\pm\omega)$  and  $A(B^\pm \rightarrow \rho^\pm\rho^0)$ . The resulting effect on the  $\pi^+\pi^-$  invariant mass distribution is shown in Fig. 1, separately for  $B^+$  and  $B^-$  decays. The vertical axis gives number of events in arbitrary units. The prominent peak, followed by a dip, is characteristic of  $\rho - \omega$  mixing [51], and does not depend strongly on the choice of hadronic parameters as long as  $|p + s|/|t + c| \ll 1$ . Note that a small CP asymmetry is expected also in case that the strong phase difference between  $s + p$  and  $t + c$  vanishes, because of the two different shapes of the  $\rho$  and  $\omega$  resonances [18, 19, 20].

In reality, limited statistics and particularly the current absence of a positive signal for  $B^0 \rightarrow \rho^0\rho^0$  would forbid carrying out the complete program leading to a construction of the pure isospin triangle (41). However, using the given invariant mass dependence of the isospin-breaking terms in (38)-(40), one may constrain these terms and eliminate them in certain cases. As noted above, a first place to look for these terms would be the  $\pi^+\pi^-$

invariant mass distribution in  $B^+ \rightarrow \rho^+ \pi^+ \pi^-$  near the  $\omega$  mass, which should be fitted to a sum of the  $\rho^+ \rho^0$  and  $\rho^+ \omega$  terms in (39) as plotted in Fig. 1.

In the relativistic Breit-Wigner form (32) we assumed no  $s$  dependence in the width  $\Gamma_V$ . This assumption has only a very slight effect on the isospin-breaking terms which are dominant at the narrow  $\omega$  peak. Non-resonant contributions could also affect the invariant mass dependence. One hopes to minimize these contributions by fitting the pion angular distributions to those describing longitudinally polarized vector meson states. Potential interference between the  $\rho$  resonance and the wide  $\rho'(1450)$  resonance must also be taken care of.

#### IV. MESON MIXING AND OTHER ISOSPIN-BREAKING EFFECTS IN $B \rightarrow \rho\pi$

The isospin method for extracting  $\alpha$  in  $B \rightarrow \rho\pi$  is based on a time-dependent Dalitz plot analysis of  $B^0 \rightarrow \pi^+ \pi^- \pi^0$  [5], using information provided by isospin symmetry [4]. We will explain now the essence of the method. Consider the contributions of  $\rho^+$ ,  $\rho^-$  and  $\rho^0$  to the amplitude of  $B^0 \rightarrow \pi^+ \pi^- \pi^0$ ,

$$A_{+-0} \equiv A(B^0 \rightarrow \pi^+ \pi^- \pi^0) = A_+ D_{\rho\rho}(s_+) \cos \theta_+ + A_- D_{\rho\rho}(s_-) \cos \theta_- + A_0 D_{\rho\rho}(s_0) \cos \theta_0, \quad (47)$$

where subscripts of the amplitudes  $A_i$ , the invariant masses  $s_i$ , and the helicity angles  $\theta_i$ , denote the charge of the  $\rho$ ,

$$A_+ \equiv A(B^0 \rightarrow \rho^+ \pi^-), \quad A_- \equiv A(B^0 \rightarrow \rho^- \pi^+), \quad A_0 \equiv A(B^0 \rightarrow \rho^0 \pi^0). \quad (48)$$

The function  $D_{\rho\rho}$  is given near the  $\rho$  pole by a Breit-Wigner form (32). Corresponding amplitudes for  $\bar{B}^0$  are denoted by  $\bar{A}$ . The time-dependent decay rate for an initially  $B^0$  is given by [52]

$$\begin{aligned} \Gamma(B^0 \rightarrow \pi^+ \pi^- \pi^0(t)) \propto & (|A_{+-0}|^2 + |\bar{A}_{-+0}|^2) + (|A_{+-0}|^2 - |\bar{A}_{-+0}|^2) \cos(\Delta mt) \\ & - 2\text{Im} (e^{-2i\beta} \bar{A}_{-+0} A_{+-0}^*) \sin(\Delta mt). \end{aligned} \quad (49)$$

The interference of the three  $\rho$  resonances in the time-dependent and invariant mass-dependent decay rate permits a determination of the magnitudes of the three amplitudes  $A_+$ ,  $A_-$ ,  $A_0$  and their charge conjugates, as well as the relative phases between these six amplitudes. This amounts to eleven independent observables, obtained from a set of twenty seven mutually dependent measurables in (49) [5]. It is convenient to rescale the  $A_i$  and  $\bar{A}_i$  amplitudes by phases  $e^{i\beta}$  and  $e^{-i\beta}$ , respectively, defining  $\mathcal{A}_i \equiv \exp(i\beta) A_i$ ,  $\bar{\mathcal{A}}_i \equiv \exp(-i\beta) \bar{A}_i$ , such that the coefficient of the  $\sin(\Delta mt)$  term in (49) measures directly the relative phases between  $\mathcal{A}_i$  and  $\bar{\mathcal{A}}_i$ .

Working in the  $t$ -convention [49], each amplitude  $A_i$  consists of a “tree” contribution proportional to  $V_{ub}^* V_{ud}$  and a “penguin” term involving  $V_{tb}^* V_{td}$ . Unitarity of the CKM matrix

is used to absorb matrix elements of penguin operators proportional to  $V_{ub}^* V_{ud}$  in the tree amplitude,

$$\mathcal{A}_{\pm,0} = e^{-i\alpha} T_{\pm,0} + P_{\pm,0} , \quad \bar{\mathcal{A}}_{\pm,0} = e^{+i\alpha} T_{\pm,0} + P_{\pm,0} , \quad (50)$$

where  $T_{\pm,0}$  and  $P_{\pm,0}$  include strong phases. The measured left-hand sides provide eleven equations for twelve unknown parameters, consisting of the weak phase  $\alpha$ , six magnitudes of tree and penguin amplitudes, and five relative strong phases between these amplitudes. An additional complex relation, leading to a total of thirteen equations for the twelve parameters, is provided by isospin symmetry. Neglecting electroweak penguin contributions and isospin breaking effects, the pure  $\Delta I = 1/2$  penguin terms vanish in the  $I = 2$  amplitude, implying

$$P_- + P_+ + 2P_0 = 0 . \quad (51)$$

The over-constrained set of equations (50), (51) allows for the extraction of  $\alpha$ . The sensitivity to  $\alpha$  can be seen explicitly by considering the  $\Delta I = 3/2$ ,  $I = 2$  amplitude  $\mathcal{A}_2$ ,

$$\mathcal{A}_2 \equiv \mathcal{A}_+ + \mathcal{A}_- + 2\mathcal{A}_0 = Te^{-i\alpha} , \quad \bar{\mathcal{A}}_2 \equiv \bar{\mathcal{A}}_+ + \bar{\mathcal{A}}_- + 2\bar{\mathcal{A}}_0 = Te^{i\alpha} , \quad (52)$$

with  $T \equiv T_+ + T_- + 2T_0$ . The angle  $\alpha$  is fixed by the relative phase between the two sums of amplitudes, which are determined up to an overall common phase by the time-dependent Dalitz plot.

The relations (51) and (52) are violated both by EWP contributions and by isospin-breaking corrections caused, for instance, by  $\pi^0 - \eta - \eta'$  mixing and  $\rho - \omega$  mixing [17]. The former contributions may be included model-independently as in the case of  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$ , using the proportionality of the  $\Delta I = 3/2$  current-current operator and the dominant  $\Delta I = 3/2$  EWP operator in the effective weak Hamiltonian [14, 53]. The relation (51) is modified to

$$P_- + P_+ + 2P_0 = P_{EW} , \quad (53)$$

where  $P_{EW}$  can be obtained from

$$\mathcal{A}_2 = Te^{-i\alpha} + P_{EW} , \quad \frac{P_{EW}}{T} = -\frac{3}{2} \left( \frac{c_9 + c_{10}}{c_1 + c_2} \right) \frac{|V_{tb}V_{td}|}{|V_{ub}V_{ud}|} = +0.013 \frac{\sin(\beta + \alpha)}{\sin \beta} . \quad (54)$$

This implies the same shift in  $\alpha$  as in  $B \rightarrow \pi\pi$ ,  $\Delta\alpha_{EWP} = (1.5 \pm 0.3)^\circ$ , if only the information from the two measured sums of amplitudes  $\mathcal{A}_2$  and  $\bar{\mathcal{A}}_2$  is used to extract  $\alpha$ . The shift in  $\alpha$  obtained from fitting the entire set (50), (53) and (54) may be slightly different, because of the additional  $\alpha$  dependence of this over-constrained system of equations.

Isospin-breaking in tree amplitudes does not affect the extracted value of  $\alpha$  in  $B \rightarrow \rho\pi$ , which is based on Eqs. (50), (53), and (54), because no isospin relation between tree amplitudes is needed. This is obvious when  $\alpha$  is extracted from (52) and is true in general. Penguin amplitudes in  $B \rightarrow \rho\pi$  are known to be small [54, 55, 56]. Therefore, other sources of isospin-breaking are expected to lead to corrections in  $\alpha$  smaller than  $\Delta\alpha_{EWP}$ , which is

related to the  $I = 2$  tree amplitude through (54). The evaluation of corrections caused by  $\pi^0 - \eta - \eta'$  mixing, that we give next, supports this expectation.

The mixing of  $\pi^0, \eta$  and  $\eta'$  introduces additional correction terms in (52),

$$\mathcal{A}_+ + \mathcal{A}_- + 2\mathcal{A}_0 = Te^{-i\alpha} + P_{EW} + \epsilon P_{\rho\eta} + \epsilon' P_{\rho\eta'} , \quad (55)$$

where  $P_{\rho\eta'}$  are penguin amplitudes in  $B^0 \rightarrow \rho^0\eta^{(')}$ . We expect that the correction to the extracted value of  $\alpha$  is smaller in  $B \rightarrow \rho\pi$  than in  $B \rightarrow \pi\pi$  because the penguin-to-tree ratio is smaller in the first case. In terms of SU(3) amplitudes defined in [50, 55], one has

$$\begin{aligned} |T| &= |t_P + t_V + c_P + c_V| , \\ P_{\rho\eta} &= \frac{1}{\sqrt{6}}(-p_P - p_V - s_V) , \\ P_{\rho\eta'} &= \frac{1}{2\sqrt{3}}(p_P + p_V + 4s_V) . \end{aligned} \quad (56)$$

A global SU(3) fit to available data of charmless  $B$  decays to a pseudoscalar and a vector meson [55] has shown that  $t_P$  and  $t_V$  add up constructively, while  $c_P$  and  $c_V$  are smaller. Also,  $|p_P/t_P| \sim |p_V/t_V| \sim 0.2$  [54, 56] and  $p_V \simeq -p_P$ , while  $s_V$  is smaller. (A best fit gives [55]  $|p_V/p_P| = 1.15 \pm 0.07$ ,  $\arg(p_V/p_P) = (182 \pm 18)^\circ$  and  $s_V/p_V = 0.16^{+0.08}_{-0.06}$ .) All this implies that the effect of the terms in (55) involving  $\epsilon$  and  $\epsilon'$  is very small. Taking

$$|T| \geq |t_V| , \quad |P_{\rho\eta}| \simeq \frac{1}{\sqrt{6}}|s_V| \leq \frac{0.3}{\sqrt{6}}|p_V| , \quad |P_{\rho\eta'}| \simeq \frac{2}{\sqrt{3}}|s_V| \leq \frac{0.6}{\sqrt{3}}|p_V| , \quad \frac{|p_V|}{|t_V|} = 0.2 , \quad (57)$$

and using  $\epsilon = 0.017 \pm 0.003, \epsilon' = 0.004 \pm 0.001$ , we find the following upper bound on the uncertainty in  $\alpha$  caused by neglecting the  $\epsilon^{(')}$  terms in (55):

$$|\Delta\alpha_{\pi-\eta-\eta'}| = \frac{|\epsilon P_{\rho\eta} + \epsilon' P_{\rho\eta'}|}{|T|} \leq 0.024\epsilon + 0.069\epsilon' \leq 0.1^\circ . \quad (58)$$

In case that the sum of  $p_P$  and  $p_V$  in  $P_{\rho\eta'}$  does not cancel completely [55], the bound could be a factor two larger. In any event, this uncertainty is much smaller than  $\Delta\alpha_{\text{EWP}}$ , the shift caused by EWP amplitudes.

Finally, we discuss the effect of  $\rho - \omega$  mixing treating it as in Sec. III. Neglecting isospin-breaking in  $g(\rho \rightarrow \pi\pi)$ , we conclude that the third term in (47) must be replaced by

$$A(B^0 \rightarrow \rho^0\pi^0)D_{\rho\rho}(s_0) \rightarrow A(B^0 \rightarrow \rho_I\pi^0)D_{\rho\rho}(s_0) + A(B^0 \rightarrow \omega_I\pi^0)\tilde{D}_{\rho\omega}(s_0) , \quad (59)$$

while the angular dependence remains unchanged. That is, the effect of  $\rho - \omega$  mixing may be included in the time-dependent Dalitz plot analysis by adding the second term in (59). The isospin-breaking function  $\tilde{D}_{\rho\omega}(s_0)$ , defined in (37) and given in (42), has a double pole structure with a narrow peak at the  $\omega$  mass. As discussed in the previous section,  $\tilde{D}_{\rho\omega}(s_0)$  is measured by studying the pion form factor, while  $A(B^0 \rightarrow \omega_I\pi^0)$  can be

obtained from  $B \rightarrow 4\pi$ . The narrow peak at the  $\omega$  mass distinguishes clearly this isospin-breaking correction from other potential non-resonant or wide resonance contributions to  $B \rightarrow \rho\pi$  [17, 57].

The size of the effect of  $\rho - \omega$  mixing may be estimated by considering the two processes,  $B^0 \rightarrow \rho^0\pi^0$  and  $B^0 \rightarrow \omega\pi^0$ , occurring in (59). The charge-averaged branching ratio of  $B \rightarrow \rho^0\pi^0$  measured by Belle is somewhat larger than an upper limit reported by BaBar, reporting also the currently strongest upper bound on  $B^0 \rightarrow \omega\pi^0$ ,

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow \rho^0\pi^0) &= \begin{cases} (5.1 \pm 1.6 \pm 0.9) \times 10^{-6}, & \text{Belle [58]}, \\ < 2.9 \times 10^{-6}, & \text{BaBar [59]}, \end{cases} \\ \mathcal{B}(B^0 \rightarrow \omega\pi^0) &< 1.2 \times 10^{-6}, \quad \text{BaBar [60]}. \end{aligned} \quad (60)$$

These values and (43) permit a relatively sizable contribution from the isospin-breaking term  $A(B^0 \rightarrow \omega_I\pi^0)\tilde{D}_{\rho\omega}(s_0)$  at the pole,  $s_0 = m_\omega^2$ , if  $\mathcal{B}(B^0 \rightarrow \omega\pi^0)$  is not much below its current upper limit [55].

## V. OTHER SOURCES OF ISOSPIN VIOLATION

In this work we have focused primarily on isospin-breaking effects in  $B \rightarrow \pi\pi$ ,  $B \rightarrow \rho\rho$  and  $B \rightarrow \rho\pi$ , originating in the mixing of neutral isospin triplet states ( $\pi^0, \rho^0$ ) with isospin singlet states ( $\eta^{(')}, \omega$ ). We also iterated the effects of higher order electroweak penguin operators. Two implicit assumptions were made in our analysis:

- Reduced matrix elements of operators in the effective Hamiltonian, between initial  $B^0$  and  $B^+$  states and final states involving  $\pi_3$  and  $\pi^+$ , were assumed to obey exact SU(2) relations.
- $\Delta I = 5/2$  corrections were assumed to vanish.

Relaxing these assumptions in  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$  introduces isospin-violating corrections in  $\alpha$  which may be comparable to those discussed in sections II and III.  $\Delta I = 5/2$  operators in  $B \rightarrow \pi\pi$  [17] or in  $B \rightarrow \rho\rho$  [8] may be induced by an insertion of the  $d - u$  mass difference  $\Delta I = 1$  operator or by electromagnetic corrections. This would violate the closure of the isospin triangles for  $B$  and  $\bar{B}$  amplitudes. Any of these other isospin-breaking corrections in  $B \rightarrow \rho\pi$  is expected to be negligible, however, because in these processes isospin breaking can only affect the relation (53) among suppressed penguin amplitudes.

To see how these other effects enter a specific calculation, let us use the result of a Soft Collinear Effective Theory approach to factorization in  $B \rightarrow M_1 M_2$ , where  $M_{1,2}$  are

pseudoscalars or vector mesons. The result, at leading order in  $\Lambda/m_B$ , is [12]:

$$A = \frac{G_F m_B^2}{\sqrt{2}} \left\{ f_{M_1} \int_0^1 du dz T_{1J}(u, z) \zeta_J^{BM_2}(z) \phi^{M_1}(u) \right. \\ \left. + f_{M_1} \zeta^{BM_2} \int_0^1 T_{1\zeta}(u) \phi^{M_1}(u) \right\} + \left\{ 1 \leftrightarrow 2 \right\} + \lambda_c^{(f)} A_{c\bar{c}}^{M_1, M_2}. \quad (61)$$

Here  $T_{iJ}(u)$  and  $T_{i\zeta}(u)$  are hard kernels which may be expanded in  $\alpha_S(m_B)$ , while  $\zeta^{BM}$ ,  $\zeta_J^{BM}(z)$  and the light cone meson wave function  $\phi^M(u)$  are nonperturbative parameters. The amplitude  $A_{c\bar{c}}^{M_1, M_2}$  denotes a possible long distance charming penguin contribution.

We have quantified isospin-breaking caused by final states which do not coincide with isospin eigenstates. The remaining isospin violation is encoded in  $\zeta^{BM}$ ,  $\zeta_J^{BM}(z)$ ,  $f_M \phi^M(u)$ , and  $A_{c\bar{c}}^{M_1, M_2}$ , where  $M_{1,2}$  are now isospin eigenstates. Generically, the corrections are expected to be of order  $(m_u - m_d)/\Lambda_{QCD} \sim \alpha_0 \sim O(1\%)$ , namely of the same magnitude as the corrections caused by  $\pi - \eta - \eta'$  mixing, Eq. (26), and by EWP contributions, Eq. (7). Isospin-breaking in the hard kernels  $T_{iJ}(u), T_{i\zeta}(u)$  occurs through additional  $1/m_B$  power-suppressed operators and may be safely neglected. At this order, no isospin violation is caused by final state rescattering in the first two terms in the amplitude (61), because these terms factorize to all orders in  $\alpha_S$  and to first order in  $\Lambda/m_B$ .

## VI. CONCLUSIONS

The extraction of the weak phase  $\alpha \equiv \phi_2$  by application of isospin symmetry to  $B \rightarrow \pi\pi$ ,  $B \rightarrow \rho\pi$  and  $B \rightarrow \rho\rho$  is modified through  $\pi^0 - \eta - \eta'$  mixing and  $\rho - \omega$  mixing. We have studied these effects in a model-independent manner, discussing also other effects of isospin-breaking in these processes. Our main results are the following:

- Isospin-breaking corrections in  $\alpha$  related to  $\pi^0 - \eta - \eta'$  mixing were bounded using flavor SU(3), and were found to be smaller than  $1.4^\circ$  in  $B \rightarrow \pi\pi$  and much smaller in  $B \rightarrow \rho\pi$ .
- The effects of  $\rho - \omega$  mixing in  $B \rightarrow \rho\rho$  and  $B \rightarrow \rho\pi$  were studied as a function of the two-pion invariant mass in terms of a quantity measured in the pion form factor. Given the invariant mass dependence characterizing  $\rho - \omega$  mixing, which involves a peak at  $s = m_\omega^2$ , we propose a way for measuring and constraining these effects experimentally. Eventually, with sufficient statistics, this procedure may eliminate the mixing effect altogether.
- In  $B \rightarrow \rho\pi$ , any kind of isospin-breaking in tree amplitudes does not affect the measurement of  $\alpha$  through a time-dependent Dalitz plot analysis. Since penguin amplitudes are suppressed, the resulting uncertainty in  $\alpha$  from isospin violation is expected to be smaller than one degree (excluding contributions from EWP operators).

- The proportionality of a  $\Delta I = 3/2$  current-current operator and a corresponding dominant electroweak penguin operator in the effective Hamiltonian implies a shift,  $\Delta\alpha_{\text{EWP}} = (1.5 \pm 0.3)^\circ$ , common to  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$ . The same shift would apply also to  $B \rightarrow \rho\pi$  if only the sums of amplitudes (52) were used. In a completely general extraction of  $\alpha$  from the time-dependent Dalitz plot fit, the shift may be slightly different but can be obtained model-independently.

A brief summary of our conclusions is therefore: (1) Isospin-breaking introduces a much smaller uncertainty in the value of  $\alpha$  extracted from  $B \rightarrow \pi\pi$  than thought before, of order  $1^\circ$ . (2) Effects of  $\rho - \omega$  mixing in  $B \rightarrow \rho\rho$  can be studied by fits to invariant mass distributions. (3) The largest shift in  $\alpha$  in  $B \rightarrow \rho\pi$ , caused by electroweak penguin amplitudes, can be included model-independently, and is about  $1^\circ$  as in  $B \rightarrow \pi\pi$  and  $B \rightarrow \rho\rho$ .

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## APPENDIX A: EXPERIMENTAL CONSTRAINTS ON $\epsilon_{1,2}$

Let us comment briefly on the values of the isospin-breaking parameters,  $\epsilon_{1,2}$ , which are constrained by fitting  $\tilde{D}_{\rho\omega}(s)$  in (42) to the pion form factor [46]. Requiring that  $D_{\rho\omega}(s)$  does not have poles at  $m_\rho^2 - im_\rho\Gamma_\rho$  and  $m_\omega^2 - im_\omega\Gamma_\omega$  implies

$$\epsilon_1 = \frac{\Pi_{\rho\omega}(m_\omega^2 - im_\omega\Gamma_\omega)}{m_\omega^2 - m_\rho^2 + i(m_\rho\Gamma_\rho - m_\omega\Gamma_\omega)}, \quad \epsilon_2 = \frac{\Pi_{\rho\omega}(m_\rho^2 - im_\rho\Gamma_\rho)}{m_\omega^2 - m_\rho^2 + i(m_\rho\Gamma_\rho - m_\omega\Gamma_\omega)}. \quad (\text{A1})$$

Using the relation

$$\tilde{\Pi}_{\rho\omega}(s) = \Pi_{\rho\omega}(s) + \frac{g(\omega_I \rightarrow \pi\pi)}{g(\rho_I \rightarrow \pi\pi)}(s - m_\rho^2 + im_\rho\Gamma_\rho), \quad (\text{A2})$$

one may express  $\epsilon_i$  in terms of the measurable function  $\tilde{\Pi}_{\rho\omega}(s)$ :

$$\begin{aligned} \epsilon_1 &= \frac{\tilde{\Pi}_{\rho\omega}(m_\omega^2 - im_\omega\Gamma_\omega)}{m_\omega^2 - m_\rho^2 + i(m_\rho\Gamma_\rho - m_\omega\Gamma_\omega)} - \frac{g(\omega_I \rightarrow \pi\pi)}{g(\rho_I \rightarrow \pi\pi)}, \\ \epsilon_2 &= \frac{\tilde{\Pi}_{\rho\omega}(m_\rho^2 - im_\rho\Gamma_\rho)}{m_\omega^2 - m_\rho^2 + i(m_\rho\Gamma_\rho - m_\omega\Gamma_\omega)}. \end{aligned} \quad (\text{A3})$$

The first term in  $\epsilon_1$  is constrained experimentally [46] (see also discussion below (42)),

$$\frac{\tilde{\Pi}_{\rho\omega}(m_\omega^2 - im_\omega\Gamma_\omega)}{m_\omega^2 - m_\rho^2 + i(m_\rho\Gamma_\rho - m_\omega\Gamma_\omega)} = (-0.003 \pm 0.002) + i(0.032 \pm 0.003) . \quad (\text{A4})$$

Barring a possible weak dependence of  $\tilde{\Pi}_{\rho\omega}$  on  $s$ , this term is equal to  $\epsilon_2$ . The term  $g(\omega \rightarrow \pi\pi)/g(\rho \rightarrow \pi\pi)$  in  $\epsilon_1$  is poorly constrained experimentally, but is expected to be of the same order. The smallness of these parameters justifies neglecting terms of order  $\epsilon_{1,2}^2$ . Note that the method presented in Sec. III for studying isospin-breaking in  $B \rightarrow \rho\rho$  depends on the function  $\tilde{\Pi}_{\rho\omega}(s)$  and not separately on its two components given in (A2).

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